Trigonometric Ratios

For GSP5

Right-triangle trigonometry builds on similar-triangle concepts to give you more ways to find unknown measures in triangles. In this activity you'll learn about trigonometric ratios and how you can use them.

SKETCH AND INVESTIGATE

In steps 1–5, you'll construct a right triangle.

1. Construct $\overline{AB}$.
2. Construct a line through point B perpendicular to $\overline{AB}$.
3. Construct $\overline{AC}$, where point C is a point on the perpendicular line.
4. Hide the line.

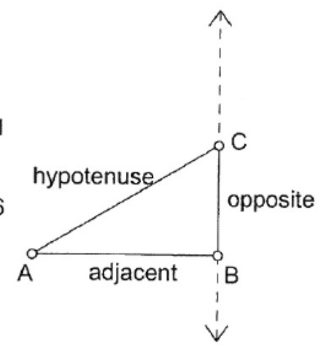
Select point *B* and $\overline{AB}$; then, in the Construct menu, choose **Perpendicular Line.**

m∠CAB = 31°

$$\frac{\text{m opposite}}{\text{m hypotenuse}} = 0.51$$

$$\frac{\text{m adjacent}}{\text{m hypotenuse}} = 0.86$$

$$\frac{\text{m opposite}}{\text{m adjacent}} = 0.60$$



5. Construct $\overline{BC}$ to finish the right triangle.
6. Show the three segments' labels and change the labels to match the figure above right.
7. Measure $\angle CAB$.
8. Measure the ratios *opposite/hypotenuse, adjacent/hypotenuse,* and *opposite/adjacent.*

Using the **Text** tool, click a segment to show its label. Double-click the label to change it.

Select, in order, points *C*, *A*, and *B*. Then, in the Measure menu, choose **Angle.**

For each ratio, select the two segments in order. Then, in the Measure menu, choose **Ratio.**

Q1 Drag point C to change the angles. When the angles change, do the ratios also change?

Q2 Drag point A or point B to scale the triangle. What do you notice about the ratios when the angles don't change? Explain why you think this happens.

Your observations in Q2 give you a useful fact about right triangles. For any right triangle with a given acute angle, each ratio of side lengths has a given value, regardless of the size of the triangle. The three ratios you measured are called *sine, cosine,* and *tangent.*

9. The sine, cosine, and tangent functions can be found on all scientific calculators, commonly abbreviated as sin, cos, and tan. Use Sketchpad's Calculator to calculate the sine, cosine, and tangent of $\angle CAB$. Match these calculations with the ratios they are equal to.

To measure an angle, press the **Marker** tool on the vertex and drag into the angle. Then select the angle marker and choose **Measure | Angle.** To measure a distance, select the two points and choose **Measure | Distance.**

Q3 Complete the ratios for cosine and tangent.

$$\text{sine } \angle A = \frac{\text{length of log opposite } \angle A}{\text{length of hypotenuse}}$$

cosine $\angle A$ = ______________

tangent $\angle A$ = ______________

Q4 Drag point C so that $\angle A$ measures as close to 30° as you can get it. Write approximate values for the sine, cosine, and tangent of 30° below. Use the definitions in Q3 and refer to the calculations in your sketch to find these values.

$\sin 30° =$ __________ $\cos 30° =$ __________ $\tan 30° =$ __________

Q5 Without measuring, figure out the measure of $\angle C$ and write down that number. Calculate the sine of that angle measure. The sine of $\angle C$ should be close to one of the trigonometric ratios for $\angle A$. Which one? Explain why this is so.

Q6 Drag point C and answer the following questions.

Hint: Make $\overline{AB}$ short so that you can drag point C up farther.

a. What's the smallest possible value for the sine of an angle in a right triangle? What angle has this value?

b. What's the greatest possible value for the sine of an angle in a right triangle? What angle has this value?

c. Why can't you make $m\angle CAB$ exactly equal to 90°?

d. Even though you can't make $m\angle CAB$ exactly equal to 90°, what do you think is the value of tan 90°? Explain.

e. For what angle is the tangent equal to 1? Why?

f. For what angle are the sine and cosine equal? Why?

g. Suppose an angle has measure x. Complete this equation:

$\sin x = \cos$ __________