Solving Systems of Equations

Name(s): ______________________

Torrance and Claire love to race. But Torrance's leg is in a cast, so on this day Claire gives him a 1/2 km head start. (They start at the same time, but Torrance starts 1/2 km ahead of Claire.) Claire runs a kilometer in four minutes, but in his cast Torrance can only walk a kilometer in nine minutes. How long will it take Claire to catch up to Torrance?

One way to solve a problem like this is to model the situation with a graph of each racer's position over time. Since Torrance's motion can be modeled with one equation and Claire's with a second, you'll be using your graph to solve two equations at the same time. When you find numbers that satisfy two or more equations at once, you're solving a *system of equations*.

Sketch and Investigate

1. In a new sketch, choose **Define Coordinate System** from the Graph menu to create a set of coordinate axes.

Click on an object with the **Text** tool to show its label. Double-click the label itself to change it.

2. Show the labels of the x- and y-axes. Then relabel the x-axis "t" (for time) and the y-axis "d" (for distance).

A good way to begin solving a word problem is to start with simple, concrete cases. The following questions and steps help you do that.

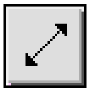
Q1 Pick an amount of time and determine how far Claire has run in that time period. Write this as an ordered pair (t, d). Let t represent time elapsed (in minutes) and d represent distance run (in kilometers).

Q2 Write a second ordered pair that represents another time and distance in Claire's run.

3. Use the **Plot Points** command from the Graph menu to plot the points corresponding to your two ordered pairs.

Q3 Which point in the sketch represents Claire's starting point?

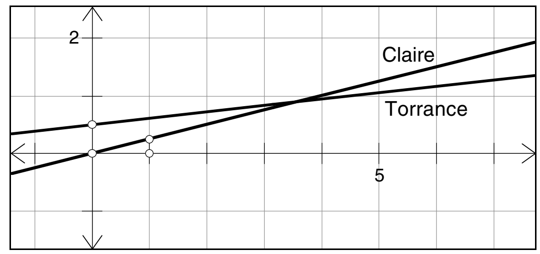
If Claire runs at a steady rate, you can model the distance she's run at any given time with a straight line. Why?



Choose the **Line** tool by pressing down on the current **Straightedge** tool and choosing the **Line** tool from the palette that pops up.

4. Use the **Line** tool to construct a line through the two plotted points. Label the line "Claire." Does the line pass through the starting point from Q3?

5. Use the same technique you used for Claire to construct a line modeling Torrance's distance from the starting line. (Don't forget to allow for his 1/2 km head start.) Label this line "Torrance."



Q4 What physical feature of the graph corresponds with the moment when Claire catches up with Torrance?

Now you'll find the exact location of the point where Claire catches up with Torrance.

6. Select both lines and choose **Intersection** from the Construct menu to find the point of intersection.

With the point selected, choose **Coordinates** from the Measure menu.

7. Measure the coordinates of the point of intersection.

Q5 How much time has passed when Claire catches up with Torrance? How far have they each gone?

Select the lines and choose **Equations** from the Measure menu.

Q6 Measure the equations of both lines. Write the system of equations that models this problem.

Q7 Explain the meaning of *each* constant in each equation in your system. For example, what does the number 0.25 in Claire's equation have to do with the problem?

Q8 Using the **Line** tool, draw a third line anywhere in your sketch. Then measure its equation. Adjust the line to plausibly model a third person's distance from the starting line. (Perhaps this person is Rollerblading.) Describe your new racer below.

Name:

Speed:

Starting point:

When and where this racer meets Torrance:

When and where this racer meets Claire:

Equation that models this racer's trip:

Present Your Findings

Make a printout showing all the race participants. Describe the three competitors. Explain the equations that model their motion, and list the coordinates of all intersection points. You may also want to add a new line modeling a horse's or bicyclist's or anyone else's motion in the race.

Explore More

1. Not every system of two linear equations has a single solution. Try to make a sketch of a system of linear equations with more than one solution, then another one with no solutions. What's true of lines that fit these criteria? Now consider systems of three linear questions, or four, or even ten! How many solutions might they have?
2. Do a circle and a line always meet in exactly two points? Describe the possible solutions to a system where one equation models a circle and the other models a line. What if both equations model circles?
3. Our race model was a good one, but not entirely accurate. Explain why the model would have been more realistic if we had used the **Ray** tool instead of the **Line** tool. Can you think of other reasons why the model wasn't very accurate?