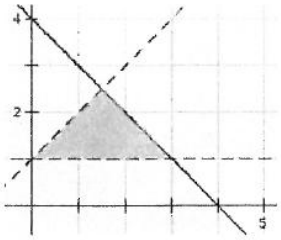
Graphing Systems of Inequalities

For GSP5

When you solve a system of equations graphically, you usually get a single point as the solution. In this activity you'll solve a system of inequalities graphically, and see what the solution of such a system looks like.



A SINGLE INEQUALITY

Begin by graphing the equation $y = \frac{3}{2}x - 2$ and the related inequality $y < \frac{3}{2}x - 2$.

1. In a new sketch, graph $y = \frac{3}{2}x - 2$ by choosing **Graph | Plot New Function** and entering the equation into the New Function dialog box.

Q1 What is the slope of the line and what are its x- and y-intercepts?

Q2 Based on the graph of the *equation*, what do you think the graph of the *inequality* will look like?

To delete the graph, select it and press the Delete or Backspace key on your keyboard.

2. Delete the function plot you just created, but leave the function itself; you will use it again in a moment.

To graph the inequality, you will use a custom tool from **Inequality Tools.gsp.**

3. Open **Inequality Tools.gsp.** Then switch back to your original sketch. (As long as **Inequality Tools.gsp** is open, you can use the tools it contains.)

Switch back to the **Arrow** tool after the graph appears.

4. Press and hold the **Custom** tool icon, and choose **Inequality Tools | y < f(x)** from the menu that appears. Click the function $f(x)$ in your sketch. A graph of the inequality appears.

You can also right-click (Windows) or control-click (Mac) the strips to get to the Properties dialog box.

5. If the graph of the inequality appears as strips, you can change the Plot Properties of this object so that it appears as a solid area. Select the strips, choose **Edit | Properties,** and use the Plot panel of the Properties dialog box to increase the number of samples. (Approximately 400 should be enough.)

Q3 Is the boundary of the inequality graph solid or dashed? Explain why its appearance makes sense in terms of the inequality.

SYSTEMS OF INEQUALITIES

Next you'll graph a second inequality ($x \geq -y + 1$) on the same coordinate system.

This function should be labeled *g*. If not, choose **Display | Label Function** to change its label to *g*.

6. Create another function by choosing **Graph | New Function.** This inequality expresses x in terms of y, so use the Calculator's Equation pop-up menu to choose **x = f(y).** Then enter the equation $g(x) = -y + 1$.

For GSP5

Q4 What do you think the graph of $x \geq -y + 1$ will look like? Write down your guess before you construct the graph.

Remember to switch back to the **Arrow** tool once the graph appears.

7. To create the graph, choose the custom tool **Inequality Tools | x >= f(y)**. Click the tool on the function $g(y)$ in your sketch.

Q5 Which shaded area contains points that satisfy both inequalities? Construct a point in that area, measure its coordinates, and use algebra to confirm that the point satisfies both inequalities.

In addition to the colors in the color menu, you can choose **Other...** to specify a different shade or tint.

Q6 For each system of inequalities below, add a new page to your document (using **File | Document Options**) and construct a graph of the system. If the graphs of two inequalities appear in the same color, change the color of one of the graphs so you can easily see the area of overlap.

a. $y \geq 2x - 1, y < -\frac{1}{2}x + 3$

b. $y < 2, x \geq -3$

c. $y \geq \frac{1}{2}x^2, x < y$

d. $y < \frac{1}{2}x^2, x \geq y - 1$

e. $x < y - 3, y \leq x - 1$

f. $y > \frac{x}{10} - x^2 + 3, y < 2$

g. $x \leq \sqrt{9 - y^2}, y > 1, x \geq -1$

h. $y < \sin x, y \geq 0.5$

For problem h, use **Edit | Preferences | Units** and change the angle units to radians.

EXPLORE MORE

Q7 Change these inequalities so that each variable appears on only one side of the inequality, making sure to keep the properties of inequality in mind. Then graph the system and describe the solution.

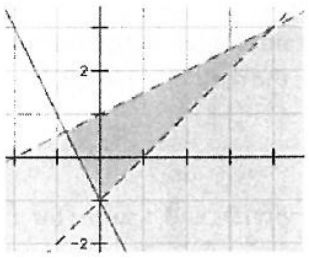
a. $2y - x < 2, |2x| \geq 2y - x$

b. $y^2 \leq x + 2y - 2, x - 3 < 0$

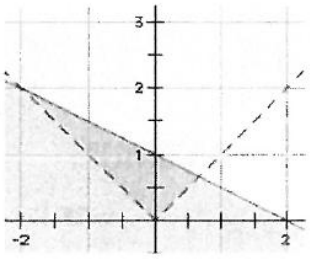
Some inequalities can be expressed with either x or y on the left side, so more than one answer may be possible for a particular graph.

Q8 Study each graph below and decide on a system of inequalities that will produce it. Check each answer on a new page of your document.

a.



b.



Q9 You probably expressed your answers to the preceding question in a form with y alone on the left side of each inequality. Where possible, rewrite your answers so that x is alone on the left side.

Q10 Use inequalities to graph an interesting shape in Sketchpad. Then challenge a friend to guess the inequalities you used.