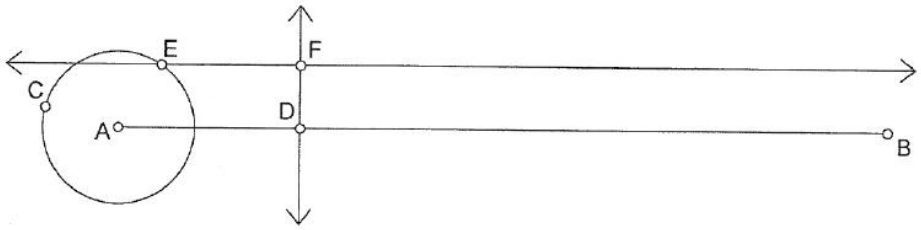
A Sine Wave Tracer

For GSP5

In this exploration you'll construct an animation "engine" that traces out a special curve called a *sine wave*. Variations of sine curves are the graphs of functions called *periodic functions*, functions that repeat themselves. The motion of a pendulum and ocean tides are examples of periodic functions.

SKETCH AND INVESTIGATE

1. Construct a horizontal segment AB.



2. Construct a circle with center A and radius endpoint C.
3. Construct point D on $\overline{AB}$.

Select point D and $\overline{AB}$; then, in the Construct menu, choose **Perpendicular Line.**

4. Construct a line perpendicular to $\overline{AB}$ through point D.
5. Construct point E on the circle.
6. Construct a line parallel to $\overline{AB}$ through point E.
7. Construct point F, the point of intersection of the vertical line through point D and the horizontal line through point E.

Don't worry, this isn't a trick question!

Q1 Drag point D and describe what happens to point F.

Q2 Drag point E around the circle and describe what point F does.

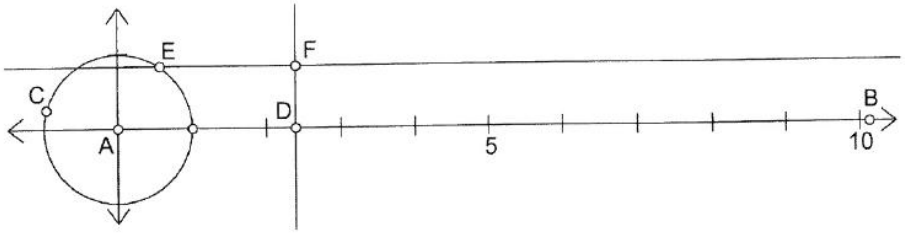
Q3 In a minute, you'll create an animation in your sketch that combines these two motions. But first try to guess what the path of point F will be when point D moves to the right along the segment at the same time that point E is moving around the circle. Sketch the path you imagine.

Select points D and E and choose **Edit | Action Buttons | Animation.** Choose **forward** in the Direction pop-up menu for point D.

8. Make an action button that animates point D forward along $\overline{AB}$ and point E forward around the circle.
9. Move point D so that it's just to the right of the circle.
10. Select point F; then, in the Display menu, choose **Trace Point.**
11. Press the Animation button.

Q4 Sketch the path traced by point *F*. Does the actual path resemble your guess in Q3? How is it different?

12. Select the circle; then, in the Graph menu, choose **Define Unit Circle.** You should get a graph with the origin at point *A*. Point *B* should lie on the *x*-axis. The *y*-coordinate of point *F* above $\overline{AB}$ is the value of the sine of $\angle EAD$.



Q5 If the circle has a radius of 1 grid unit, what is its circumference in grid units? (Calculate this yourself; don't use Sketchpad to measure it because Sketchpad will measure in inches or centimeters, not grid units.)

13. Measure the coordinates of point *B*.

14. Adjust the segment and the circle until you can make the curve trace back on itself instead of drawing a new curve every time. (Keep point *B* on the *x*-axis.)

Q6 What's the relationship between the *x*-coordinate of point *B* and the circumference of the circle (in grid units)? Explain why you think this is so.