Radian Measure

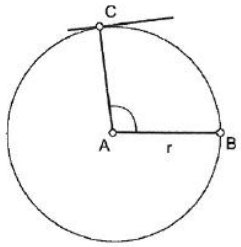
For GSP5

So you think you know angle measurement? An understanding of degrees is a valuable skill, but there are other ways to measure. Other angle units include points, grads, mils, and dekans. To make matters worse, these units may go by different names in different places. Still worse, they may have the same name, but different definitions.

Among all of these angle units, the radian holds a special place. You can use it to measure angles, of course, but radian measure also describes relationships between certain geometric objects.

WHAT IS A RADIAN?

1. Open **Radian Measure.gsp.** Press the *Go* button and watch the circle radius rotate into a tangent position and then roll around the circle.



You will use this line segment to measure a central angle of the circle. This will be the basis for defining radian measure for angles.

2. Press *Reset* and then *Home* to stop the animation and return the radius to its tangent position. Measure the radius of the circle and the length of the blue segment.
3. Press the buttons *Show Central Angle, Show Arc,* and *1 Radian.* Measure the length of arc *a.*

Q1 The central angle, $\angle BAC$, is now exactly one radian. What do you notice about your three measurements? Explain why they come out this way.

Notice that although θ is equivalent to $m\angle BAC$, it can keep increasing past 360°.

Q2 The measure of the angle, θ, is displayed in degrees. Approximately how many degrees are there in one radian?

4. Press *Semicircle.* The line segment will continue to roll until it has stepped off half of the circle.

Q3 Using the tick marks to approximate an answer, how many radians are in a semicircle? How many radians will there be in a complete circle?

5. Press the *1 Circle* button to check your last answer.
6. Choose **Edit | Preferences.** Change the Angle Units to **radians.**

Q4 The angle measurement now shows you exactly how many radians are in a circle. But you already knew that, didn't you? Write the formula for the circumference in terms of the radius. Use that along with the definition of a radian to prove that there are exactly 2π radians in a circle.

WHY RADIANS?

So far, you have not seen any good reason for using radians rather than degrees. Actually, we use radians in order to make things easier, not harder.

Do these measurements one at a time. Select one object and choose the appropriate command from the Measure menu.

7. Press *Reset* and *Go*. Press *Go* again to stop the animation before the angle makes a complete circle ($0 < \theta < 2\pi$).

8. You have a measurement for angle θ and a measurement for radius r. Use the calculator to find the product $\theta \cdot r$.

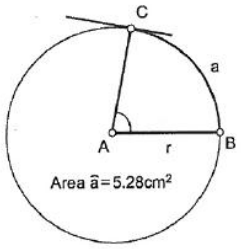
To change the radius, drag point *B*.

Q5 What is the arc length in terms of θ and r? Check your answer with different radii and different angles in the range $0 < \theta < 2\pi$. Does your formula always work? Does it work when you use degrees?

The area of a circle sector varies directly with the central angle. You probably are familiar with this formula:

$$\text{sector area} = \frac{\theta}{360^\circ}\pi r^2$$

Q6 Rewrite the above formula using radians instead of degrees. Simplify your answer.



9. Select the arc and choose **Construct | Arc Interior | Arc Sector.** Select the sector and choose **Measure | Area.**

Q7 Using your formula from Q6, calculate the area of the sector. Does it match your measurement in all cases?

DISCUSS

Q8 When using radians, Sketchpad automatically expresses angle measurements in multiples of π. This is a common practice. Why?

Q9 It is also common practice (not used by Sketchpad) to write radian angle measurements without writing any units at all. Why is that?

Q10 In spite of the radian advantages you have seen here, degrees are more common in practical applications. What advantages do degrees have?