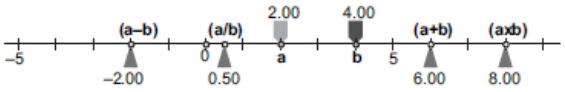
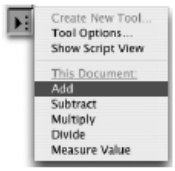
Exploring Properties of Operations

In this activity you'll use Sketchpad arithmetic machines to investigate the properties of the four fundamental arithmetic operations.



INVESTIGATE



1. Open **Operation Properties.gsp.**
2. On the Addition page, use the **Add** custom tool to construct a marker showing the sum of *a* and *b*.
3. On a copy of the Operation Properties chart, make the title of the chart "Operation Properties for Addition," and fill in the blanks in the "Property" column with addition signs.
4. For each row of the chart, experiment by dragging *a* and *b* to determine whether that row's property is possible. If the property is not possible, write "Never" and explain why in the "When is it true?" column. If possible, fill in three examples, and write a sentence in the "When is it true?" column describing the conditions that must be met for the description to be true. Be specific about the values that the two numbers can have.
5. When you finish with one operation, go to the next page of the sketch, and use the appropriate custom tool to construct a result marker on that page. Then fill out the Operation Properties chart for that operation.

Here's an example of what you might write in the first row of the Addition chart:

Property	Examples	When is it true?
$a + b = 0$	$a = 5, \quad b = -5$ $a = -3, \quad b = 3$ $a = 0, \quad b = 0$	The sum of two numbers is zero when the numbers are the opposites of each other (or both equal zero).

PRESENT

You can use the Help system to learn how to make and use Movement, Animation, and Hide/Show buttons.

On the Presentation page of the document, choose one particular property of one of the operations, and create a presentation sketch that uses Movement, Animation, or Hide/Show buttons to demonstrate the circumstances in which the property is true.

Operation Properties for _____

Property	Examples	When is it true?
$a \square b = 0$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a \square b = 1$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a = b = a \square b$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a = a \square b$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a > 0$, $b > 0$, and $a \square b > 0$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a < 0$, $b < 0$, and $a \square b < 0$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a \square b > a$ and $a \square b > b$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a \square b$ is between a and b	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	
$a \square b < a$ and $a \square b < b$	$a =$ $b =$ $a =$ $b =$ $a =$ $b =$	

Geometric Descriptions of Algebraic Properties

You can use this chart to help you figure out what geometric features to look for when you investigate a particular algebraic property. For the algebraic property in each row of the table, write a sentence describing the corresponding geometric behavior of the markers for a, b, and $a \otimes b$. Row 7 is filled in as an example. ($\otimes$ stands for $+$, $-$, $\cdot$, or $\div$.)

Row	Algebraic Property	Geometric Description
1	$a \otimes b = 0$	
2	$a \otimes b = 1$	
3	$a = b = a \otimes b$	
4	$a = a \otimes b$	
5	$a > 0$, $b > 0$, and $a \otimes b > 0$	
6	$a < 0$, $b < 0$, and $a \otimes b < 0$	
7	$a \otimes b > a$ and $a \otimes b > b$	The $a \otimes b$ marker is to the right of both the a and the b marker.
8	$a \otimes b$ is between a and b	
9	$a \otimes b < a$ and $a \otimes b < b$	