Ellipses and Hyperbolas

There are many different methods that can be used to create ellipses and parabolas. The method that we will use here is closely related to a method that involves drawing a circle on wax paper selecting a point outside (for the hyperbola) or inside (for the ellipse) the circle and folding and folding such that the point corresponds with a point on the circle. This is repeated several times until an ellipse or hyperbola appears.

SKETCH AND INVESTIGATE

1. Begin with a new sketch. Use the Compass Tool to create a circle. Label the center of the circle *A*, and the point on the circle, *B*. Place another point on the circle and label it *C*.
2. Place a point outside the circle and label the point *D*.
3. Select points *C* and *D* and choose Segment from the Construct menu.
4. Select segment *CD* and choose Midpoint from the Construct menu. Label the midpoint *E*.
5. Select midpoint *E* and segment *CD* and choose Perpendicular Line from the Construct menu.
6. Select the perpendicular line and then choose Trace Perpendicular Line from the Display menu.
7. Drag point C around the circle.

Q1. What do you notice about the traces formed by the perpendicular lines as you drag point C around the circle? Why does this make sense?

8. Select Erase Traces from the Display menu. Click on the perpendicular bisector and click Trace Perpendicular Line from Display menu.
9. Use the Line tool to create Line AC
10. Select Line AC and the perendicular line through point E and choose Intersection from the Construct menu.
11. Label this point F. Choose point F and select Trace Intersection from the Display menu.
12. Drag point C around the circle.

Q2. What do you notice about the traces formed by the point as you drag point C around the circle? Why does this make sense?

13. Select point A and point F and choose Distance from the Measure menu.
14. Select point D and point F and choose Distance from the Measure menu.
15. Choose Calculate from the Number menu. Find abs(AF-DF).
16. Drag point C.

Q3. What do you notice about the difference between AF and DF? Why does this make sense.

Q2. What will happen if you drag point D so that it is further from the center of the circle?

17. Select point D and drag it. Choose Erase Traces from the Display menu. Drag point C around the circle. What do you notice?
18. Rather than trace the line we can also create the locus that will update when point D is moved. Select the perpendicular line and point C. Choose Locus from the Construct menu.

Q3. Drag point D so that it is near the circle and inside the circle. What happens to the locus? Use similar methods as you did in exploring the hyperbola to explain what is happening to the locus when point D is inside the circle.