To show how the lengths of fencing and the areas of the animal pens are related, you may have written an equation equivalent to $A = \frac{L^2}{4\pi}$, or perhaps you wrote an equation equivalent to $L = 2\sqrt{\pi A}$, where L stands for the length of fencing and A stands for the area of the enclosed circular animal pen. You could also have made a table showing some possible lengths and corresponding areas. Perhaps you even sketched a graph. How are these ways of showing relationships between varying quantities related to the specific way that the concept of function is defined?

Definitions of *function*

Function is defined in many ways. Consider the selection of informal and formal definitions of *function* from middle school, high school, and college-level mathematics textbooks in table 1.1.

Table 1.1
Textbook definitions of *function*

A	A function is a relationship between input and output. In a function, the output depends on the input. There is exactly one output for each input.
B	A function is a relation in which each element of the domain is paired with *exactly* one element of the range.
C	A function is a set of ordered pairs (or number pairs) that satisfies this condition: There are no two ordered pairs with the same input and different outputs.
D	A real-valued function f defined on a set D of real numbers is a rule that assigns to each number x in D exactly one real number, denoted by f (x).
E	A function is a rule that assigns to each element of a set *A* a unique element of a set *B* (where *B* may or may not equal *A*).
F	For any sets *A* and *B*, a function f from A to B, f: A → B, is a subset *f* of the Cartesian product *A* × *B* such that every $a \in A$ appears once and only once as the first element of an ordered pair (*a*, *b*) in *f*.
G	A function is a mapping or correspondence between one set called the domain and a second set called the range such that for every member of the domain there corresponds exactly one member in the range.
H	One quantity, *H*, is a function of another, *t*, if each value of *t* has a unique value of *H* associated with it. We say *H* is the *value* of the function or the *dependent variable*, and *t* is the *argument* or *independent variable*. Alternatively, think of *t* as the *input* and *H* as the *output*.

Sources: Definitions A and B, Holliday et al. (2005), pp. 43 and 226, respectively.
Definition C, Interactive Mathematics Program (2000), p. 5.
Definition D, Edwards and Penney (2002), p. 2.
Definitions E and F, Usiskin et al. (2003), pp. 68 and 70.
Definition G, Saxon (2003), p. 152.
Definition H, Hughes-Hallet et al. (1994), p. 2.

Note: Italics as in originals; definition D defines *real-valued* functions only.