Solving Compound Inequalities

A *simple inequality* declares one condition that must be true. For example, to satisfy the inequality $2x < 3 + x$, $2x$ must be less than $3 + x$. A *compound inequality* declares two or more conditions, and some combination of them must be true. For this activity, we will stick to two inequalities.

SUBSTITUTION ON THE NUMBER LINE

1. Open **Compound Inequalities.gsp.**

Point x is attached to the number line, and its coordinate is used to evaluate the two inequalities above the number line:

$$2x < 6 - x \qquad 7x + 10 > x - 8$$

Q1 Drag point x along the number line. A line segment sometimes appears above the points, and another one sometimes appears below. For each line segment, what determines whether it will appear?

2. Select both line segments. Choose **Display | Trace Segments.**

3. Slowly drag point x to trace the solutions of both inequalities.

Q2 What is the solution set of the left (red) inequality?

Q3 What is the solution set of the right (blue) inequality?

Q4 Consider this compound inequality:

$$2x < 6 - x \quad \text{or} \quad 7x + 10 > x - 8$$

When the word "or" is used, the solution is the *union* of the two sets.

The word "or" indicates that the solution includes all values of x for which one or both inequalities are true. What is the solution set of this compound inequality?

Q5 This is a different compound inequality:

$$2x < 6 - x \quad \text{and} \quad 7x + 10 > x - 8$$

When the word "and" is used, the solution is the *intersection* of the two sets.

The word "and" indicates that the solution includes only the values of x for which both inequalities are true. What is the solution set of this compound inequality?

Q6 Model the compound inequalities on the next page, and report their solution sets. To change the direction of an inequality sign, click the button above the sign. To edit one of the four expressions, double-click it; to enter x into the calculation, click the measurement x in the sketch. After you set the inequality signs and expressions correctly, press the *Erase Traces* button, and drag x to see the new trace.

a. $13 - x > -5x - 15$ or $4x + 10 > 17 - 3x$

b. $13 - x > -5x - 15$ and $4x + 10 > 17 - 3x$

c. $30 - 13x > 30 - 7x$ or $8x + 18 < 11x$

d. $30 - 13x > 30 - 7x$ and $8x + 18 < 11x$

SYMBOLIC SOLUTIONS

The method you have been using amounts to guess-and-check. It's more reliable and often more efficient to solve both inequalities symbolically by undoing operations, and then to compare the solution sets.

4. Page 2 shows a number line plot of the compound inequalities $x \geq -4$ and $x < 2$. Notice that the inequality is already solved.

Q7 Why is the circle at -4 filled while the one at 2 is open?

5. Take a few minutes to experiment with the objects on this page. Press the buttons and edit the small numbers above the inequalities.

Q8 To solve the compound inequalities below, solve the parts separately and combine the solutions using the sketch. In each case, report the solution set and sketch a number line plot.

a. $6x < 9x - 30$ or $4x + 12 > -x + 2$

b. $5x \leq 8x + 24$ or $7x - 10 < 2x + 15$

c. $5x - 9 > 12 - 2x$ and $3x + 13 \leq 45 - x$

THE INEQUALITY GAME

The One Inequality page shows an inequality (or sometimes an equation). Press the *Play* button to change it. Then work the graph out by yourself and press the *Show* button to check your answer.

Go to the Compound Inequality page. Press the *and/or* button to choose which type of compound inequality to use.

The One Graph and Compound Graph pages provide the same games in reverse: You see the graph and must derive the expression. On the Compound Graph page, if your answer does not match the one on the screen, check again. You may be right. There is often more than one expression that will produce the same graph.