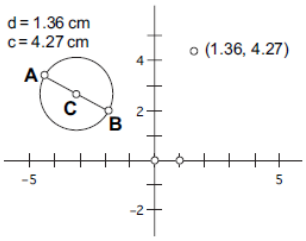
# The Circumference Function

For GSP5

In a *function,* one quantity depends on another quantity, just as the number of songs a jukebox plays depends on how much money is put in. In the case of a circle, we can say that its circumference *depends* on its diameter—the farther it is across a circle, the farther it is around it. In this activity you'll explore this connection as a *functional* relationship between two changing quantities.

## IMAGINE AND PREDICT

Imagine using a compass to draw a circle. You measure the circle's diameter (1.36 cm) and calculate its circumference (4.27 cm). You then plot the point (1.36, 4.27) on a piece of graph paper. You draw a second circle, measure its diameter ($d$), calculate its circumference ($c$), and plot a second point ($d$, $c$). You do this for many more circles.




**Q1** What will your graph look like after you plot many points? Discuss this with your group. Will the graph be straight or curved? Why? If it's straight, what will its slope be? If it's curved, will it bend up or down? Will it go forever in both directions, or will it start or stop somewhere? Write down your predictions.

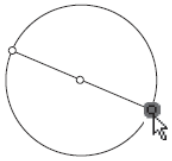
## SKETCH AND INVESTIGATE


The **Segment** tool


The **Compass** tool

Now test your prediction using Sketchpad.

1. In a new sketch, construct a segment using the **Segment** tool. With the segment selected, choose **Construct | Midpoint.**
2. Construct a circle whose diameter is the segment. To do this, choose the **Compass** tool and click first on the midpoint and then on one endpoint. Drag each endpoint of the diameter to make sure your construction holds together.



3. Measure the segment's length by selecting it and choosing **Measure | Length.** Similarly, select the circle and measure its circumference.

Double-click a measurement with the **Text** tool (the *A* in the Toolbox) to edit its label.

4. Label the diameter measurement $d$ and the circumference measurement $c$.
5. Plot the point ($d$, $c$) by selecting the measurements in order and choosing **Graph | Plot as (x, y).** The plotted point appears.

You can drag the tick mark numbers on the axes to change the scale of the graph.

6. Select the plotted point and choose **Display | Trace Plotted Point.**

**Q2** The moment of truth has arrived. Drag either endpoint and observe the point being traced. Describe this trace on your paper. Was your prediction correct?

## FURTHER INVESTIGATION

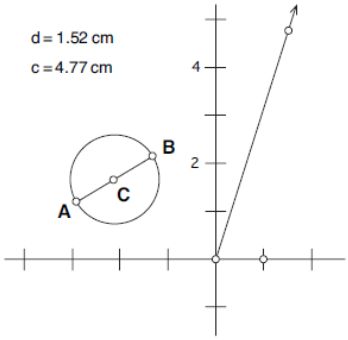
You've now seen the shape of the diameter-circumference trace. But why does it look the way it does? You may know that lines can be modeled with equations of the form $y = mx + b$, or $c = md + b$ in this case. So what are $m$ and $b$ for this line?

**Q3** Why does it make sense that the trace goes through the origin? What does this tell you about $m$ and $b$ in the equation $c = md + b$?

**Q4** Drag the diameter endpoints so that $d$ is as close to 1.00 as you can make it. What is the approximate circumference of the circle? Is this number familiar to you? If so, what's it called?

Choose **Number | Calculate** to open the Calculator. Click *c* and *d* in the sketch to enter them into the calculation.

**Q5** Use Sketchpad's Calculator to find the ratio $c/d$. Drag one of the diameter endpoints and observe the ratio. What happens and why?




7. Turn off tracing for your plotted point.
8. Construct a ray from the origin through the plotted point. Then measure the ray's slope.

**Q6** What does the slope measurement tell you about $m$ and $b$ in $c = md + b$?

You've now seen the same number in three places:

- the circumference of a circle with diameter 1
- the ratio of any circle's circumference to its diameter
- the slope of the graph of a circle's circumference as a function of its diameter

Choose **Graph | Plot New Function** and enter your proposed expression. Use *x* for the diameter (because you used *d* for the *x*-coordinate earlier).

**Q7** Use your results for $m$ and $b$ from Q3 and Q6 to plot a line that includes the ray from step 8. On your paper, write the function you used.

## EXPLORE MORE

**Q8** What's an appropriate domain for the circumference function? Select the function plot from Q7 and choose **Properties** from the Edit menu. Go to the Plot panel and set an appropriate domain for this situation.

**Q9** Consider a circle's *area* as a function of its radius. How will the plot of that function compare with the diameter-circumference plot? Make a prediction, and then use the techniques from this activity to confirm.