SKETCH AND INVESTIGATE

Q1 The perpendicular bisector of any chord of a circle passes through the center of the circle.

Q2 The closer the chord is to the center of the circle, the longer the chord. The chord is longest when its distance to the center is zero. This chord is a diameter of the circle.

Q3 At the point where the locus intersects the *y*-axis, the length of $\overline{BC}$ is zero (its minimum value) and the length of $\overline{AD}$ is the radius of the circle (its maximum value). Likewise, at the point where the locus intersects the *x*-axis, the length of $\overline{BC}$ is the diameter of the circle (its maximum value) and the length of $\overline{AD}$ is zero (its minimum value).

As point *G* moves from left to right, its *y*-coordinate decreases in value, showing the chord $\overline{BC}$ moving closer to the center of the circle (and also becoming longer).

Students may notice that the locus shows a portion of an ellipse in the first quadrant. This ellipse is centered at the origin and has a major axis of twice the diameter of the circle and a minor axis of twice the radius of the circle.

Q4 If two chords in a circle are congruent, the chords are the same distance from the center of the circle. (The converse is also true.)

EXPLORE MORE

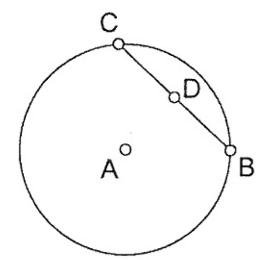
1. When $HI = BC$, the plotted point (length of $\overline{HI}$, distance from $\overline{HI}$ to the center) is coincident with point *G*.
2. After constructing the arc, construct two chords anywhere on the arc. Construct the perpendicular bisectors of both these chords. Their point of intersection is the center of the circle.

Chords in a Circle

For GSP5

A chord in a circle is a segment with endpoints on the circle. In this activity you'll investigate properties of chords.

SKETCH AND INVESTIGATE



1. Construct circle *AB*.
2. Construct chord *BC*.
3. Construct the midpoint *D* of the chord.

Select point *D* and $\overline{BC}$; then, in the Construct menu, choose **Perpendicular Line.**

4. Construct a line through point *D* perpendicular to $\overline{BC}$. This line is the perpendicular bisector of the chord. (It's not shown in the figure.)
5. Drag point *C* around the circle and observe the perpendicular line.

Q1 Write a conjecture about the perpendicular bisector of any chord in a circle.

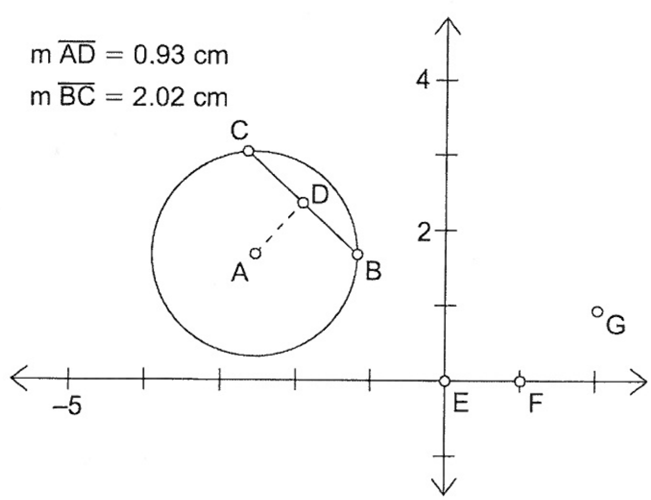
Select the line; then, in the Display menu, choose **Hide Line.**

6. Hide the perpendicular bisector and construct $\overline{AD}$.
7. Measure the length of $\overline{AD}$. This is the distance from the chord to the center.
8. Measure the length of $\overline{BC}$.
9. Drag point *C* around the circle and observe the measures.

Q2 How is the length of the chord related to its distance from the center?

If you don't see point *G*, scale the axes by dragging point *F* toward point *E*.

10. You can make a graph that shows this relationship: Select the length of $\overline{BC}$ and the length of $\overline{AD}$, in that order; then, in the Graph menu, choose **Plot As (x, y)**. You should get axes and a point *G* whose coordinates are the measures you selected.

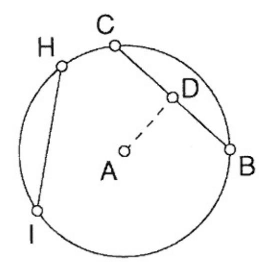


11. Drag point *C* to see how it controls point *G*.
12. To graph all the possible locations for point *G*, select it and point *C*; then, in the Construct menu, choose **Locus**.
13. Drag point *C* to see point *G* travel along the locus.
14. Drag point *A* or point *B* to see what effect changing the circle's radius has on the graph.

Q3 Write a paragraph describing the graph. Answer these questions in your paragraph: Look at the value of *y* where the locus intersects the *y*-axis. What does this value represent in the circle? Look at the value of *x* where the locus intersects the *x*-axis. What does this value represent in the circle? As point *G* moves from left to right, what happens to the value of its *y*-coordinate? What does this have to do with what's happening to the chord? Use a separate sheet of paper.

15. Construct $\overline{HI}$, another chord on the circle.
16. Measure *HI*.
17. Measure the distance from $\overline{HI}$ to the center of the circle.
18. Drag point *H* or point *I* and watch the length measure. Try to make this length as close to the length of $\overline{BC}$ as you can.

Select $\overline{HI}$ and point *A*; then, in the Measure menu, choose **Distance.**



Q4 Write a conjecture about congruent chords in a circle.

EXPLORE MORE

1. Plot as (x, y) the length of $\overline{HI}$ and the distance from $\overline{HI}$ to the center. How does this plotted point compare to point *G* when $HI = BC$?
2. An arc is part of a circle. You can construct an arc from any three points. In a new sketch, construct a three-point arc. Now use your conjecture from Q1 to construct the center of the circle containing the arc. Construct the circle to confirm that you found the correct point. Explain what you did.