The 30°-60° Right Triangle

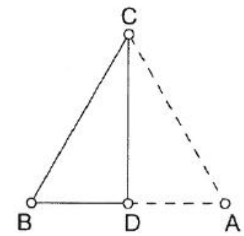
For GSP5

The 30°-60° right triangle—formed by taking half of an equilateral triangle—has special relationships among its side lengths. These relationships make it easy to find all the side lengths if you know just one. In this activity you'll discover these relationships and why they hold.

SKETCH AND INVESTIGATE

You can use the tool **3/Triangle (By Edge)** from the sketch **Polygons.gsp**. If you need to relabel the triangle, use the **Text** tool. Click a point to show its label. Double-click a label to change it.

1. In a new sketch, construct an equilateral triangle *ABC*. Use a custom tool or construct it from scratch. Drag vertices to confirm that the construction is correct.



2. Hide the triangle's interior if necessary.
3. Construct the midpoint *D* of $\overline{AB}$.
4. Construct median *CD*.

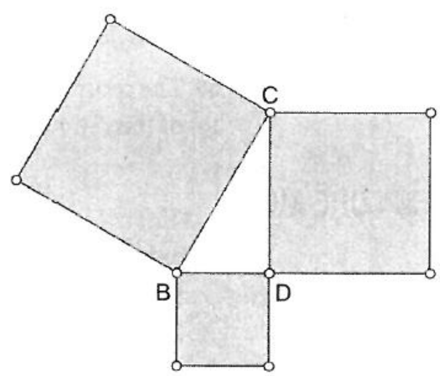
Select the segments; then, in the Display menu, choose **Line Style | Dashed.**

5. Change the line styles of $\overline{AB}$ and $\overline{AC}$ to dashed.
6. Construct $\overline{BD}$ and make its line style thin.

Q1 Without measuring, state the measure of each angle in $\triangle CDB$. For each angle, explain how you know it has that measure.

Q2 In a 30°-60° right triangle, how does the length of the hypotenuse compare to the length of the short leg? Answer without measuring.

7. Hide point *A*, $\overline{AB}$, and $\overline{AC}$.
8. Use a custom tool to construct squares on the three sides of the triangle as shown at right. Drag to make sure the squares are properly attached.



9. Measure the areas of the three squares.

Choose **Calculate** from the Number menu to open the Calculator. Click a measurement to enter it into a calculation.

10. Calculate the ratio of the largest area to the smallest area
11. Drag point *B* and observe this ratio.

Q3 What is this area ratio? Explain why this ratio is what it is. In your explanation, use what you know about the side lengths.

Q4 Now you'll use your answer to Q3 about the square on the hypotenuse and the square on the short leg to help you find the area of the square on the long leg. Answer the following questions.

a. Suppose the smallest square has area x^2. What would be the area of the square on the hypotenuse? ____________

b. Use the Pythagorean theorem to find the area of the square on the long leg. Show your work.

c. State the ratio of the area of the square on the long leg to the area of the square on the short leg. Calculate this ratio in your sketch and drag point *B* to confirm that this ratio applies to all 30°-60° right triangles. ____________

Q5 Suppose the short leg had length *x*.

a. What would be the length of the hypotenuse? ____________

b. What would be the length of the long leg? ____________

c. What would be the ratio of the length of the long leg to the length of the short leg? (State your answer in radical form.) ____________

Select the hypotenuse and the short leg. Then, in the Measure menu, choose **Ratio.** Measure the other ratio in the same way.

12. To confirm your answers to Q5, measure the ratio of the hypotenuse length to the short leg length. Also measure the ratio of the long leg length to the short leg length. Drag point *B* to confirm that these ratios apply to all 30°-60° right triangles.

Q6 The second ratio you measured in step 12 should be the decimal approximation of the ratio you wrote in Q5c. Record this decimal approximation. ____________

EXPLORE MORE

13. To test how well you can apply your discoveries, make a Hide/Show action button for each side length measurement. Show one side length and hide the other two. Then calculate the two hidden lengths. Show the hidden lengths to check your calculations. Try this several times, changing the triangle each time and showing different side lengths. Repeat until you think you can calculate the missing side lengths correctly every time.